Disclosure Policies in All-pay Auctions with Bid Caps and Stochastic Entry

Written by
Bo Chen, KFUPM
Lijun Ma, SZU
Yu Zhou, HKUST
Zhubo Zhao, SZU

August 8, 2020

Unknown Number of Competitors

Many competitions feature unknown number of competitors:

- In a job promotion, individuals may complete with anonymous candidates from outside labor market.
- In R&D races, firms do not know the actual number of R&D race competitors.
- When players buy lottery tickets, they do not know the actual number of players
- ……

Bid Caps

Many competitions also feature enforced bid caps:

- U.S. Federal law limits both congressional election campaign contributions and spending.
- In job promotion, candidates cannot work more than 24h per day.
- The Chinese government enforced bid caps in land auctions.
- ……

Research Questions

- How does a bidder behave differently when he does not the exact number of competitors he will face?
- What are the implications for the expected total bid or effort?
- Would contest organizer fully concealing the number of bidders, or fully revealing it?

We build a model in the spirit of Che and Gale (1998) (an all-pay auction with exogenous bid cap) to study the optimal disclosure policy for contest organizers.

- Departure: exogenous stochastic entry

Summary

- Two effects arise when the number of participants become overt with an existence of bid caps:
  - (Friction effect) restricts the highest bid when the number of participants turns out to be low.
    - $\downarrow$ efforts
  - (Competition effect) incentivizes bidders to shift their median-level efforts to equal bid caps when the number of participants turns out to be high.
    - $\uparrow$ efforts
- If the contest organizer can choose the disclosure policy, she prefers to fully conceal the number of bidders.

The Literature

- Optimal disclosure policy in competitions.
  - Lim and Matros (2009), Fu et al. (2011), Chen et al. (2017)
  - McAfee and McMillan (1987), Feng and Lu (2016)
  - Our paper: unobservable numbers of competitors, effort domain restrictions
- Effects of bid caps
  - Che and Gale (1998, 2006), Szech (2015)
  - Gavious et al. (2002), Olszewski and Siegel (2019)
  - Our paper: optimal disclosure policies

Model Setup

- Three dates: $t = \{1, 2, 3\}$. n potential risk neutral bidders with paticipation probability p. One indivisible prize.
  - $t = 1$, the contest organizer commits to reveal or conceal and announce a bid cap h.
  - $t = 2$, nature chooses the number of participating bidders, organizer learns this number m, and participating bidders submit their bids b.
  - $t = 3$, the one with the highest bid wins the prize, and ties are resolved by fair lotteries.
- Bidders' realized payoffs are:

$$W_i = \begin{cases} 1 - b_i & \text{if } b_i > \max_{j \in M \setminus \{i\}} b_j \\ -b_i & \text{if } b_i < \max_{j \in M \setminus \{i\}} b_j \\ \dfrac{1}{\#\{k \in M : b_k = b_i\}} - b_i & \text{if } b_i = \max_{j \in M \setminus \{i\}} b_j \end{cases}$$

Full Concealment

We focus on mixed-strategy symmetric equilibrium: all bidders submit bids following same distribution of bids $F(x)$ ($F_m(x)$). An equilibrium is characterized by $\{F(x), F_m(x), c, c_m, h\}$.

Proposition (Full Concealment)

Consider the subgame that follows policy C. The unique symmetric equilibrium in which each bidder's equilibrium distribution of bids is given by

$$F(x) = \begin{cases} \left[[x + (1-p)^{n-1}]^{1/(n-1)} - (1-p) \right] / p & \textit{for } x \in [0, c] \\ \left[[c + (1-p)^{n-1}]^{1/(n-1)} - (1-p) \right] / p & \textit{for } x \in (c, h) \\ 1 & \textit{for } x = h \end{cases}$$

where the critical value $c = c(h)$ is defined by

if $h \leq \frac{1-(1-p)^n}{np} - (1-p)^{n-1}$, $c = 0$;

if $h \in (\frac{1-(1-p)^n}{np} - (1-p)^{n-1}, 1 - (1-p)^{n-1}]$,

$h = \frac{1-[c+(1-p)^{n-1}]^{n/(n-1)}}{n\left[1-[c+(1-p)^{n-1}]^{1/(n-1)}\right]} - (1-p)^{n-1}$.

Full Concealment

Proposition (Full Concealment con't)

The expected payment of a participating bidder is

$$EP^C = \begin{cases} h & \text{if } h \leq \frac{1-(1-p)^n}{np} - (1-p)^{n-1} \\ \dfrac{1-(1-p)^n}{np} - (1-p)^{n-1} & \text{if } h \in \left(\frac{1-(1-p)^n}{np} - (1-p)^{n-1}, 1-(1-p)^{n-1} \right] \end{cases}$$

Full Revealing

Proposition (Full Revealing)

Consider the subgame that follows policy D. If there is $m = 1$ participating bidder, the only participating bidder will bid 0. Consider a contest among $m \geq 2$ bidders. In the unique symmetric equilibrium, each bidder's equilibrium distribution of bids is given by

$$F_m(x) = \begin{cases} x^{1/(m-1)} & \text{for } x \in [0, c_m] \\ c_m^{1/(m-1)} & \text{for } x \in (c_m, h) \\ 1 & \text{for } x = h \end{cases}$$

where the critical value $c_m = c_m(h)$ is defined by

$$\begin{aligned} &\text{if } h \leq 1/m, c_m = 0; \\ &\text{if } h \in (1/m, 1], h = \frac{1 - c_m^{m/(m-1)}}{m[1 - c_m^{1/(m-1)}]}. \end{aligned}$$

The expected payment of a participating bidder is

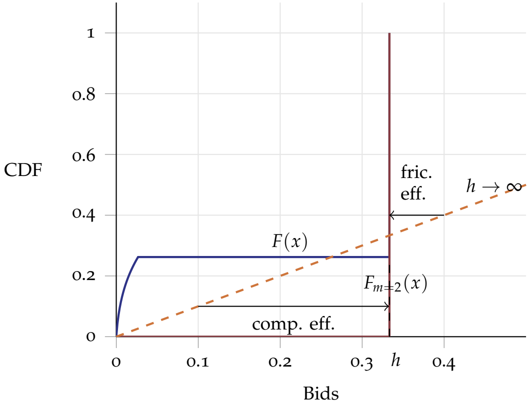
$$EP_m = \begin{cases} h & \text{if } h \leq 1/m \\ 1/m & \text{if } h \in (1/m, 1]) \end{cases}$$

Revenue Ranking

Revenue Ranking

If $h \geq 1/2$, the expected total bid is the same under the two disclosure policies. If $h \in (0, 1/2)$, the expected total bid is higher under full concealment.

Intuition:



The effects over bidding strategy:

- low m ⇒ bid more aggressively
- cap blocks the highest bid ⇒ $b \downarrow$ (friction effect)
- capped maximal bid ⇒ median level bids jump equal to cap ⇒ $b \uparrow$ (competition effect)

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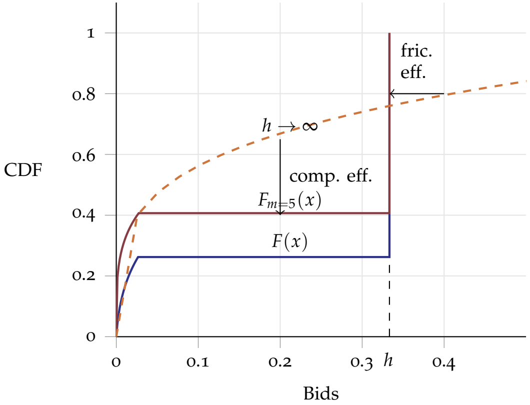

The effects over bidding strategy:

- high $m \Rightarrow$ bid less aggressively
- cap blocks the highest bid $\Rightarrow b \downarrow$ (friction effect)
- capped maximal bid $\Rightarrow$ median level bids jump equal to cap $\Rightarrow b \uparrow$ (competition effect)

Conclusion

- Two strategic effects brought by a restrictive bid cap when considering organizers' optimal disclosure policies
  - Friction effect $\Rightarrow b \downarrow$
  - Competition effect $\Rightarrow b \uparrow$
- Friction effects dominates.
- Organizers prefer fully concealing the information about the number of participating bidders.